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Spacetime Foam

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Abstract

Spacetime is composed of a fluctuating arrangement of bubbles or loops called spacetime foam, or quantum foam. We use the holographic principle to deduce its structure, and show that the result is consistent with gedanken experiments involving spacetime measurements. We propose to use laser-based atom interferometry techniques to look for spacetime fluctuations. Our analysis makes it clear that the physics of quantum foam is inextricably linked to that of black holes. A negative experimental result, therefore, might have important ramifications for semiclassical gravity and black hole physics.

Essay on Gravitation

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Spacetime appears smooth on large scales. On small scales, however, it is bubbly and foamy due to quantum fluctuations. In this essay, we use the holographic principle to show that the fluctuations are much larger than what conventional wisdom leads us to believe. We alternatively derive the same results by carrying out gedanken experiments to measure distances and time intervals. Intriguingly, the fluctuations are large enough that they may soon be detectable with modern laser-based atom interferometers.

From the holographic principle to spacetime foam

The holographic principle grew out of the profound insights of Wheeler, Bekenstein, Hawking, 't Hooft, and Susskind. [1] It states that the maximum number of degrees of freedom that can be put into a region of space is given by the area of the region in Planck units. To connect it to quantum foam, let us consider a region of space measuring $R \times R \times R$, and imgine partitioning it into cubes as small as physical laws allow. Into each small cube we put one degree of freedom. If the smallest uncertainty in measuring a distance R is δR, in other words, if the fluctuation in distance R is δR, then the smallest such cubes have volume $(\delta R)^3$. (Otherwise, one could divide R into units each measuring less than δR, and by counting the number of such units in R, one would be able to measure R to within an uncertainty smaller than δR.) Thus the maximum number of degrees of freedom, given by the number of small cubes we can put into the region of space, is $(R/\delta R)^3$. The holographic principle demands that $(R/\delta R)^3 \lesssim (R/l_P)^2$, where $l_P = ct_P \equiv (\hbar G/c^3)^{1/2}$ is the Planck length. This yields

$$\delta R \gtrsim (R l_P^2)^{1/3} = l_P \left(\frac{R}{l_P}\right)^{1/3}. \tag{1}$$

Thus quantum fluctuations from individual bubbles of spacetime add together to produce a (curious) $\sqrt[3]{R}$-dependence and are much larger than the folklore [2] indicates (viz., $\delta R \gtrsim l_P$). The corresponding metric fluctuation is given by $\delta g_{\mu\nu} \gtrsim (l_P/R)^{2/3}$. Note that even for a macroscopic distance R, the fluctuation δR, though much larger than the Planck scale l_P, is still incredibly small; e.g., for $R = 1$ km, δR is to an atom as an atom is to a human being. Since the holographic principle is deeply rooted in black hole physics, this way of deriving spacetime fluctuations is highly suggestive of the deep connection between quantum foam and black hole physics.

From spacetime measurements to spacetime foam

For an alternative means of deriving δR, let us consider a gedanken experiment to measure the distance R between two points. The need for carrying out explicit measurements to determine distances is implicit in general relativity, according to which, coordinates do not have any meaning independent of observations; in fact, a coordinate system is defined only by explicitly carrying out spacetime distance measurements. Following Wigner [3], we can put a clock at one of the points and a mirror at the other. By sending a light signal from the clock to the mirror in a timing experiment, we can determine the distance. However, the quantum uncertainty in the positions of the clock and the mirror introduces an inaccuracy δR in the distance measurement. Let us concentrate on the clock and denote its mass by m. Wigner argued that if it has a linear spread δR when the light signal leaves the clock, then its position spread grows to $\delta R(2R/c) = \delta R + \hbar R(mc\delta R)^{-1}$ when the light signal returns to the clock, with the minimum at $\delta R = (\hbar R/mc)^{1/2}$. Hence one concludes that

$$\delta R^2 \gtrsim \frac{\hbar R}{mc}. \tag{2}$$

One can supplement this quantum mechanical relation with a limit from general relativity [4]. To see this, let the clock be a light-clock consisting of two parallel mirrors (each of mass $m/2$), a distance l apart, between which bounces a beam of light. For the uncertainty in distance measurement not to be greater than δR, the clock must tick off time fast enough that $l/c \lesssim \delta R/c$. But l, the size of the clock, must be larger than the Schwarzschild radius Gm/c^2 of the mirrors, for otherwise one cannot read the time registered on the clock. From these two requirements, it follows that

$$\delta R \gtrsim \frac{Gm}{c^2}, \tag{3}$$

the product of which with Eq. (2) yields precisely the expression for spacetime fluctuation δR given by Eq. (1), obtained by using the holographic principle. (Conversely, we can argue that the holographic principle has its origin in the quantum fluctuations of spacetime.) A gedanken experiment to measure a time interval T gives an analogous expression:

$$\delta T \gtrsim (T t_P^2)^{1/3}. \tag{4}$$

Interrelationship between spacetime foam and black hole physics

It is interesting that the gedanken experiment used above to deduce the structure of spacetime foam can also be applied to discuss the precision and the lifetime of a clock. For a clock of mass m, if the smallest time interval that it is capable of resolving is t and its total running time is T, we find $t^2 \gtrsim \frac{\hbar T}{mc^2}$, [5] the analogue of Eq. (2), and $t \gtrsim \frac{Gm}{c^3}$, the analogue of Eq. (3).* If one uses a black hole of mass m as a clock, with t given by the light travel time across the black hole's horizon, i.e., $t \sim \frac{Gm}{c^3}$, then one immediately finds that $T \sim \frac{G^2 m^3}{\hbar c^4}$, which is just Hawking's black hole lifetime! Thus, if we had not known of black hole evaporation, this remarkable result would have implied that there is a maximum lifetime (of this magnitude) for a black hole. This is another demonstration of the intimate (if, in this case, indirect) relationship between quantum foam and black hole physics.

One can also translate the clock relations into useful expressions for a simple computer. The fastest possible processing frequency is obviously given by t^{-1}. Thus we identify $\nu = t^{-1}$ as the clock rate of the computer, i.e., the number of operations per bit per unit time. The identification of the number I of bits of information in the memory space of a simple computer is subtler. Since T/t is the maximum number of steps of information processing,

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*One can combine these two expressions to give $T/t^3 \lesssim t_P^{-2} = \frac{c^5}{\hbar G}$, which relates clock precision to its lifetime. [5] (Note that this new expression is just Eq. (4) with $\delta T \to t$.)

we tentatively make the identification $T/t \rightarrow I$.† If a black hole (of mass m) is viewed as an information-processing system, then its memory space has $I = T/t \sim (m/m_P)^2$, where $m_P \equiv (\hbar c/G)^{1/2}$ is the Planck mass. This gives the number of bits I as the event horizon area in Planck units, as expected from the identification [1] of a black hole entropy! Furthermore, the number of operations per unit time for a black hole computer is given by $I\nu \sim mc^2/\hbar$, in agreement with Lloyd's results [6] for the ultimate physical limits to computation. All these results indicate the conceptual interconnections of the physics underlying simple clocks, simple computers, black holes, and spacetime foam.

Interferometers as detectors of spacetime foam

Now we come to an important question: how do we detect quantum foam, i.e., how do we check Eq. (1) and Eq. (4) experimentally? It has been suggested that modern gravitational-wave interferometers can potentially provide a way, because the intrinsic foaminess of spacetime gives another source of noise in the gravitational-wave interferometers that can be highly constrained. [7] Here we propose to use a smaller and simpler experimental setup, and optimistically suggest that laser-based atom interferometry experiments [8] will be precise enough in the not-too-distant future to detect spacetime fluctuations on the scales of quantum gravity at the level given by Eq. (1) and Eq. (4). In a laser-based atom interferometer, an atomic beam is split by laser beams into two coherent wave packets which are kept apart before being recombined by laser beams. The phase change of each wave packet is proportional to the proper time along its path, and so the resulting interference pattern depends on the time difference between the two paths. In the absence of spacetime fluctuations, the phase change η over a time interval T is given by $\eta(T) = \Omega T$, where $\Omega \equiv mc^2/\hbar$ is the quantum angular frequency associated with the mass m of the atom. Due to spacetime fluctuations (Eq. (4)), there is an additonal fluctuating phase $\delta\eta$ given by

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†Using these identifications, one gets $I\nu^2 \lesssim \frac{c^5}{\hbar G}$. This expression links together our concepts of information, gravity, and quantum uncertainty. [5]

$$\delta\eta \sim \frac{(Tt_P^2)^{1/3}}{T}\eta = (Tt_P^2)^{1/3}\Omega. \tag{5}$$

For example, in 1992, Chu and Kasevich at Stanford University built an atom interferometer which used sodium atoms ($m \sim 4.5 \times 10^{-26}$ kg), and the two wave packets were kept apart for 0.2 sec. [9] For that experiment, one finds that $\eta(T) \sim 7 \times 10^{24}$ radians and $\delta\eta \sim 3 \times 10^{-4}$ radians. Thus one needs a precision of about 1 part in 10^{29} to look for spacetime foam (through suppression of the interference pattern), compared with the precision of 1 part in 10^{26} that was then achieved. In other words, one needs a (mere) thousandfold improvement in noise sensitivity to detect spacetime fluctuations.

In summary, we have combined the general principles of quantum mechanics and general relativity to address the problem of quantum fluctuations of spacetime. A simple application of the holographic principle has shown that spacetime undergoes much larger quantum fluctuations than one may expect. This result is confirmed by gedanken experiments for spacetime measurements. We believe that the Planck scale, so far only a hypothetical extreme regime, will eventually become a realm that can be approached and measured, for instance by interferometry techniques. In this essay, we have also highlighted the interconnection between spacetime foam and black hole physics. Hence, if future experiments show that spacetime fluctuates at a level smaller than our prediction $\delta R \sim \sqrt[3]{Rl_P^2}$, we will know that our current understanding of semiclassical gravity and black hole physics may need a drastic revision. We hope these arguments are sufficiently convincing to encourage a determined experimental quest to detect the very fabric of spacetime, for, as Faraday wrote:

> *Nothing is too wonderful to be true, if it be consistent with the laws of nature, and in such things as these, experiment is the best test of such consistency.*

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